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Концепция рекурсии в когнитивных исследованиях. Часть II: От Тьюринга к Байесу и к сознанию

Михайлов Игорь Феликсович,

доктор философских наук,
ведущий научный сотрудник,
Институт философии РАН
Москва, Россия

ifmikhailov@iph.ras.ru

Аннотация. В статье обсуждается концепция рекурсии в математике, ИИ, когнитивных исследованиях и ее связь с сознанием. Развитие понятия прослеживается параллельно с историей теории вычислимости, когда были введены концепции машины Тьюринга с оракулом и вероятностной машины Тьюринга. Также рассматриваются такие рекурсивные вычислительные методы, как байесовская рекурсивная оценка и байесовский иерархический вывод. Показано, что с каждым нововведением в рекурсивных методах пределы вычислимости расширялись. Автор утверждает, что рекурсия является жизненно важным аспектом человеческого познания, особенно в разработке и интерпретации сложного языка. В статье также характеризуются проблемы изучения рекурсии и сознания, такие как субъективная природа сознания и сложность нейронных сетей, связанных с сознательным мышлением. Кроме того, анализируются ограничения в понимании рекурсии современных теорий когнитивной обработки, освоения языка и сознания. Делается вывод о том, что исследование связи между рекурсией и сознанием имеет решающее значение для развития более глубокого понимания языка и когнитивной обработки. Автор ожидает, что будущая теория, основанная на рекурсии, поможет решить основные метафизические загадки прошлого и настоящего.

Ключевые слова: рекурсия; когнитивная обработка; сознание; усвоение языка; нейронные сети.

The Concept of Recursion in Cognitive Studies. Part II: From Turing to Bayes to Consciousness

Mikhailov Igor Felixovich,
Doctor of Philosophy, Leading researcher,
Institute of Philosophy of the Russian Academy of Sciences
Moscow, Russian Federation

ifmikhailov@iph.ras.ru

Abstract. This article discusses the concept of recursion in mathematics, AI, cognitive studies and its relationship to consciousness. The development of the notion is followed in parallel with the history of computability theory when concepts of Turing oracle and probabilistic machines were introduced. Also, such recursive computational techniques as Bayesian Recursive Estimation and Bayesian hierarchical inference are reviewed. It is shown that, with each novation in recursive methods, the limits of computability have expanded. The author argues that recursion is a vital aspect of human cognition, particularly in the development and interpretation of complex language. The paper also addresses the challenges of studying recursion and consciousness, such as the subjective nature of consciousness and the complexity of neural networks associated with conscious thought. Additionally, the paper examines the limitations of current theories of cognitive processing and language acquisition in understanding recursion and consciousness. The article concludes that investigating the relationship between recursion and consciousness is critical for developing a deeper understanding of language and cognitive processing. The author anticipates that a future recursion-based theory will help solve principal metaphysical conundrums of the past and the present.

Keywords: recursion; cognitive processing; consciousness; language acquisition; neural networks.

> Philosophy is written in this all-encompassing book that is constantly open before our eyes, that is the universe; but it cannot be understood unless one first learns to understand the language and knows the characters in which it is written.
>
> *Galileo Galilei 'The Assayer'*

> The freedom of the will consists in the impossibility of knowing actions that still lie in the future.

We could know them only if causality were an *inner* necessity like that of logical inference.

Ludwig Wittgenstein 'Tractatus Logico-Philosophicus'

The evolution of Turing computability up to post- and non-Turing models

The brief history of the concept of recursion and that of recursiveness of a function in their relation to the concepts of computation and computability was outlined in [1]. In what follows I will try to show that the present day of cognitive science is shaped by the meeting of general recursive (computational) approach therein with what may be labelled as the probabilistic turn in sciences of mind, most of all represented in various Bayesian approaches both in neurobiology and in Machine Learning. Taking into consideration Soare's recommendation to 'distinguish between the intensional meaning of Church's Thesis (that all effectively calculable functions are general recursive) versus Turing's Thesis (that all intuitively computable functions are computable by a Turing machine) [italics omitted — I. M.]' [2, p. 313], yet I will go on adhering to the extensional equality of the recursive and the computable, unless it undermines the overall idea of the paper.

As Soare reminds in [3, p. xxi–xxii], Turing introduced the concept of *oracle machines* in a one-page description in his dissertation, consisting of a Turing machine connected to an 'oracle' for querying during computation, which he compares to a modern local server connected to a large database. Emil Post expanded this concept and defined what he called *Turing reducibility*. A set B is Turing reducible to a set A ($B \leq_T A$) if there is an oracle machine that can compute B when the characteristic function of A is written on the oracle tape. This idea laid the foundation of modern computability theory. The notion helped define that of *Turing equivalence* as follows: two sets are Turing equivalent if each one is Turing reducible to the other. This is an equivalence relation, and it partitions the set of all sets of natural numbers into Turing equivalence classes. Turing equivalence can be used to compare the relative information content of non-computable (undecidable) sets. If sets A and B are Turing equivalent (i.e., $A \leq_T B$ and $B \leq_T A$), we view A and B as encoding the same information. Turing reducibility gives us a precise measure of the information they encode relative to other sets. The *Tur-*

ing degrees of these sets are equivalence classes containing sets with the same information content. Thus, the concept of Turing equivalence provides a precise and useful measure of the amount of information encoded in a non-computable set.

In [2, p. 302-303] Soare discusses the history of relative computability. The concept of relative computability is concerned with studying the relationship between two objects when one is computable relative to the other. Kleene and Post developed the concept of oracle Turing machines further by studying the computations that can be performed relative to an oracle. This led to the development of the theory of *degrees of unsolvability*, which is concerned with classifying computational problems based on their degree of difficulty relative to an oracle. In particular, Kleene and Post showed that there exist oracles such that the Turing machines relative to those oracles can compute functions that are uncomputable by any ordinary Turing machine.

In [4], under discussion is the problem of providing human-readable and understandable explanations of the decisions made by decision support systems using Bayesian networks, particularly in the context of the most probable explanation (MAP) problem. The concept of MAP-independence is introduced as a means of capturing the idea of relevance, or whether an unobserved variable might potentially impact the explanation. The computational complexity of several problems based on this concept is examined, particularly by addressing issues of specific kinds of Turing machines with an oracle. Oracle machines are instrumental in approaching computability of Bayesian inferences, specifically in the context of assessing the computational complexity of maximizing MAP-independence in Bayesian networks [4, p. 20].

Adding an oracle to a standard Turing machine as a solution to some otherwise intractable problems is also discussed in [5] in the context of modelling the ‘autopoietic’ human interaction with environment.

It seems significant for the topic of the probabilistic turn in cognitive science that the probabilistic Turing machine [6, p. 128-130] is often interpreted as a TM with a random oracle [7, p. 7]. The question of whether subsequent extensions of the classical (Turing) computability may help extend the class of computable functions is both historically and theoretically discussed in [8]. The same authors propose their own contribution to the required update of the classical construction in the form of Interactive Computation Theory [9]. It posits a simple model of interactive computing which uses a single component and an environment that interact using input and output signals. The article describes a simple model of interactive computing which uses a single component and an environment that interact using input and output signals. The model builds on the theory of ω-automata and characterizes interactive computation in a stream setting.

The interactive computation model builds on the theory of ω-automata and characterizes interactive computation in a stream setting. ω-automata are machines that operate on infinite sequences of input. In the interactive model, the input is produced by the environment and, correspondingly, the output of the program is a sequence of actions in response to the input. However, the behaviour of the program is not determined purely by the input sequence, but also by the program state that changes over time as a result of its interaction with the environment.

The model of interactive computation is purely deterministic, as everything inside the system runs deterministically. The only non-deterministic aspects are any unpredictable time-delays that may occur due to external factors, such as the speed of the environment or input/output channels. But, as has been shown in Part I and will be confirmed further, empirically, probabilistic models of agent-environment interaction prove to be more realistic to the day.

Having said this, let us put our problem this way: if cognitive functions (say, language) are recursive, does it imply that the correlating neural functions (say, spike exchange) are recursive too? In Part I, this superstructure is casually labelled 'meta-recursion', which is not to be identified with the far more technical notion of *metarecursion* (a.k.a. *higher recursion*) as introduced by Gerald E. Sacks in [10; 11]. Probably, a better term could be 'hyper-recursion' or 'super-recursion', but let us admit that our intuitions on that matter has not yet matured enough to claim for a strict terminology.

Superstructures of recursion

As for language, there is an interesting study [12], that suggests that recursion might help the child acquire conceptual understanding of propositional attitudes and false beliefs. They argue that sentences are the appropriate medium for attitude concepts and claim that it is false complements rather than recursion per se that act as the ultimate trigger for the understanding of false beliefs. This is supported by ongoing language acquisition experiments. Children necessarily acquire sentential complementation prior to acquiring the understanding of false beliefs, and truth contrasts enable children to recognize recursion more easily. This gives the child a way to represent false belief statements, which in turn leads to the understanding of false beliefs. Thus, to process propositions like 'The girl said there was a butterfly in her hair (but it is a leaf)' in order to be able to give the right answer to the question 'What did the girl say there was in her hair?' — 'a butterfly' instead of 'a leaf' — a child starts with detecting truth contrasts between clauses 'the girl said' (true) and 'there was a butterfly' (false) to proceed with both recursive complements in syntax and propositional attitudes in semantics.

In view of our aims, one can easily suggest that linguistic recursion is an ability acquired via learning, but it is made possible by recurrent loops in the brain. This relation may be characterized as that of weak supervenience, when a change in the 'upper' system necessarily correlates with *any* change in the 'lower' one, but *not the particular* change. This reservation is due to the brain plasticity and multi-functionality. Some intuitions into the brain's recurrent structure and its computational performance may be obtained from studying recurrent artificial neural networks.

In [13] the computational and dynamical capabilities of biological neural networks are under discussion via comparison of the computational powers of diverse theoretical neural models with those of abstract computing devices. The authors suggest that some intrinsic computational capabilities of the brain might lie beyond the scope of Turing-equivalent models of computation, surpassing the potentialities of all current standard artificial models of computation.

They concluded that the computational powers of recurrent neural networks involved in the interactive computational framework follow similar patterns of characterization as those involved in a classical computational framework. The results showed that the incorporation of either evolving capabilities or some power of the continuum in a basic neural model provides an alternative and equivalent way towards the achievement of super-Turing computational capabilities. These achievements suggest that some intrinsic computational capabilities of the brain might lie beyond the scope of Turing-equivalent models of computation.

Recurrent neural networks are capable of processing dynamical information, which has been shown to be relevant to brain dynamics, such as chaotic properties that cannot be described by the universal Turing machine model. Moreover, when equipped with real-valued weights, recurrent neural networks become capable of super-Turing computation, hence powerful enough to solve some hardest computable functions in polynomial time. These results were obtained by the authors through the so-called Thesis of Analog Computation, which states that no reasonable abstract analog device can be more powerful than first-order analog recurrent neural networks.

Reminiscent of the famous Marvin Minsky's metaphor of 'Society of Mind' is the fact that the power of interactive probabilistic computations reveals itself not only in neural networks, but in multi-agent systems as well. There is an interesting discussion of a framework for multi-agent reinforcement learning that adopts variational Bayes methods to approximate the opponents' conditional policies, to which each agent finds the best response and then improves their own policies [14]. Similar to the paper examined above, the reasoning studied here is called recursive reasoning because it represents the belief reasoning process where each agent considers the reasoning process of other agents, based on which it expects to make better decisions. Importantly, it allows an opponent to reason about the modelling agent rather than being a fixed type; the process can therefore be nested in a form as "I believe that you believe that I believe …". Recursive reasoning is similar to the way that humans think about reasoning. Despite some initial trails, there has been little work that tries to adopt this idea into the multi-agent deep reinforcement learning setting. The methods are tested on both the matrix game and the differential game, and they are proved to converge in the self-play scenarios when there exists one Nash equilibrium.

Variational Bayes methods are used to approximate a complex probability distribution with a simpler distribution that is easier to calculate. In the context of multi-agent reinforcement learning, it is used to approximate the opponents' conditional policy. This approximation is then used by each agent to find the best response and improve their own policy. The paper discusses the framework of multi-agent reinforcement learning with action class and continuous state space. In this framework, each agent uses a deep neural network to approximate its policy and the value function. Moreover, to decentralize the learning process, they use the advantage actor-critic algorithm and the independent network learning algorithm. Finally, they use the variational Bayes method to approximate the opponents' conditional policy.

In this context, 'conditional' refers to the probability distribution of the action of one agent conditioned on the state of the environment and the policy of the other agent(s). The opponents' conditional policy indicates how they would behave based on the current state of the environment and their own policies. The variational Bayes method is used to approximate this conditional policy.

In the experiments discussed in the paper, the proposed method is compared with other existing methods on two different benchmarks: one with continuous action space, and the other with discrete action space. The results show that the proposed method outperforms the existing ones on both benchmarks. Moreover, the authors also show the ablation study, where they remove the relevance of each component of the proposed framework in turn and compare the resultant methods. They argue that each component is critical to the overall performance of the algorithm.

In this study, "action space" refers to the set of all possible actions that an agent can take in a given environment. In a multi-agent reinforcement learning problem, the action space of each agent is dependent on the state of the environment and the policies of other agents. In the paper, the authors use two different benchmarks, one with continuous action space and the other with discrete action space.

General cognitive recursion

Michael Corballis is the author of numerous works developing the framework of the 'recursive mind'. In his comprehensive book [15] he presents the mathematical concept of recursion by providing examples such as the factorial, Fibonacci sequence, and the Kanji script, and describes how recursion can extend indefinitely in theory but is limited in practice. Recursion is defined as a process that repeats itself and solves problems by breaking them down into simpler versions of the same problem. The author emphasizes that recursion gives rise to the concept of infinity, a mental achievement that is unique to humans.

He also discusses the features that distinguish human language from other forms of animal communication, specifically the concept of recursion. Recursion, the ability to embed structures within structures in a recursive fashion, allows humans to generate an infinite number of sentences. The author describes various forms of recursion, including center-embedded recursion, and provides examples of its use in language. He also debates whether animals have language and defines language as a uniquely human trait. Finally, the author discusses the evolution of language, and how it evolved from manual gestures to vocalizations, before becoming the complex system of communication that it is today.

In Part 2 called 'Mental Time Travel', Corballis discusses the ability of humans to mentally transport themselves to other places and times, specifically through the power of memory and imagination. He introduces the concept of mental time travel, a phenomenon that allows humans to imagine and relive previous experiences, as well as plan possible future events. The author then connects mental time travel to language, suggesting that the ability to remember, plan, and tell stories may have been a critical factor in the evolution of human language and social cohesion. He provides several specific examples of mental time travel in everyday life, along with the role that storytelling and fiction play in this process.

In Part 3 — 'Theory of Mind', the author discusses how humans are able to understand what others are thinking or feeling, and introduces the concept of mind-reading, not as a psychic ability but as a natural ability to infer the mental perspectives of other people. He suggests that this ability is critical to social cohesion and cooperation and explains how theory of mind was also critical to the emergence of language. The chapter describes specific experiments that demonstrate the importance of theory of mind in social interactions, and how it develops in childhood. Additionally, the author discusses the limitations that may exist in our understanding of the thoughts and feelings of others, and the role that culture and socialization play in shaping our theory of mind abilities.

Corballis argues that recursion plays a critical role in the development of Theory of Mind. Recursive thinking allows individuals to infer not only what others are thinking or feeling but also to infer what others might think about what they are thinking or feeling. This capacity is essential for social interaction and communication, especially in the form of language. For example, in a conversation, people have to understand not only what the other person is saying but also what they mean by it, which requires recursive thinking. The author notes that theory of mind, like language, may have evolved from earlier cognitive capacities that were not language-based but that possessed recursive properties that allowed language to emerge.

Bayesian Brain

The growing family of Bayesian approaches in AI and other cognitive sciences uses various computational techniques to model cognition through anticipation. Of a principal interest for the present study is the method known as Recursive Bayesian Estimation. According to [16], RBE is an approach used in statistics and machine learning that estimates the current state of a system. It is used in robotics and automotive technology to perform a task that requires estimation. For example, a self-driving car can estimate its location using the Recursive Bayesian Estimation framework. The car obtains its starting position via a Global Positioning System (GPS), and its built-in algorithms then help it estimate its current location after a certain amount of time or distance. These algorithms often use the Bayesian mathematical concept in statistics.

The two essential concepts of Recursive Bayesian Estimation are recursive estimation and Bayesian inference. Recursive estimation refers to the process of estimating the current state based on the previous state, where the new state estimation will become the basis for the subsequent state estimation. Bayesian inference, on the other hand, makes it possible to obtain the state of an entity based on the prior state of an entity and the likelihood function or prediction.

Recursive Bayesian Estimation can be applied to other fields outside of statistics and machine learning. RBE applications range from the automotive industry to medical science. For example, the remaining useful life of medical devices can be predicted using recursive Bayesian estimation. Industrial robots are also designed to estimate their position using this estimation method. In general, recursive Bayesian estimation is a flexible mathematical approach that has helped advance technologies in many fields beyond statistics and machine learning.

Daunizeau in [17] explores a Bayesian approach to understanding the evolution of cognition in the brain. The author highlights the need to understand the principles of optimality in the brain to understand its functions. The concept also suggests that information theory offers a probabilistic account of how data is optimally processed, and Bayes' rule is crucial to understanding the brain.

The Bayesian Brain hypothesis identifies optimal solutions to the computational problems that the brain faces, promoting distinct properties such as precision, efficiency, flexibility, and sophistication. It takes a formal perspective on how selective pressure may shape the brain's information processing mechanisms. It can be described on three levels of abstraction: the computational level, algorithmic level, and the physical/implementational level. It bridges the gap between the first two levels of analysis by describing how information processing should operate for realizing a given cognitive function.

Information theory provides a probabilistic account of how information is optimally transmitted and processed by relying on probability calculus and, more precisely, on Bayes' rule. It was originally developed to address problems of numerical signal processing such as data compressing and storage. It quickly extended beyond academic barriers, finding applications in statistics, artificial intelligence, physics, and telecommunications. More recently, information theory was successfully applied to model the way the brain perceives, understands and/or learns about its environment. Therefore, it helps explain how the brain perceives, learns, and understands information by providing a formal and consistent framework for introducing optimality principles into cognitive science research.

Optimality principles can help us understand the brain's mechanisms better. In cognitive science research, optimality principles are used to identify the best solution to a given computational problem that the brain faces while performing a cognitive function. Optimal solutions are defined in a formal and quantifiable manner, allowing researchers to make clear predictions about the expected performance of the cognitive system under different conditions. Furthermore, by introducing optimality principles into cognitive science research, it becomes possible to test whether the brain actually achieves the predicted performance or not, and in case it doesn't, whether it is due to the presence of a competing constraint or to some suboptimality in its mechanism.

The Bayesian Brain hypothesis is closely related to the concept of recursion. In general, recursion is a computational structure that allows the formulation of complex problems in terms of simpler ones. In other words, it allows for the chaining of a simple cognitive process to generate more complex behaviours. Similarly, the Bayesian Brain hypothesis proposes that cognitive functions at different levels of complexity can be understood in terms of Bayesian inference of probabilistic models. Given this, the Bayesian Brain framework can provide explanations of the cognitive processes that underlie recursive computation in perception, learning, and decision-making. In this sense, it constitutes a formal account of the type of recursive computation observed in the brain.

Recursion in brain computations is related to Bayesian hierarchical inference. Bayesian hierarchical inference refers to the process of inferring the structure of a complex generative model given a set of observed data. It involves assigning probabilities to the parameters and the hyperparameters of the model and using these probabilities to make inferences about the model's structure. Recursion refers to the computational structure of breaking down complex problems into simpler ones that can be understood as elementary building blocks, which can be further recursively nested to generate higher-order complexity. In the Bayesian Brain framework, the use of Bayesian hierarchical models enables the modeling of complex connectivity patterns at multiple levels of the nervous system. Using this approach, the brain is construed as generating more and more complex cognitive representations that exhibit the same type of recursive computational structure found in Bayesian hierarchical inference. Therefore, recursion in brain computations is related to Bayesian hierarchical inference, and this relationship provides a powerful means of explaining the brain's learning and computational mechanisms.

The Bayesian Brain Hypothesis is related to *Predictive Processing Theory*. PPT offers a bottom-up approach to perception, where top-down predictions about future sensory input modify the processing of incoming information. These predictions are based on generative models that implicitly encode statistical regularities within the external and internal environment. In contrast, the Bayesian Brain hypothesis provides a top-down approach to cognition, where cognitive processes are seen as implementing Bayes' rule to estimate the most likely cause of sensory input. Both models share the same Bayesian framework, which makes it possible to integrate sensory input and internal models to construct accurate perceptual representations of the world. In this sense, Bayesian models of the brain can provide a general theoretical framework for predictive processing, while PPT provides a mechanistic account of the precise neural computations that support Bayesian inference.

Recursive consciousness

According to the algorithmic information theory of consciousness [18], the existence and use of compressive models by cognitive systems in biological recurrent neural networks enables and provides the structure to phenomenal experience, which includes self-awareness as a part of a better model. Recursion allows these cognitive systems to interact bidirectionally with the external world, giving rise to apparently complex (entropic but hierarchically organized) data. This theory is called KT, based on the mathematical theory of Kolmogorov complexity, which provides a natural framework to study and quantify consciousness from neurophysiological or neuroimaging data, given the premise that the primary role of the brain is information processing.

According to KT, the human brain creates models of its sensory inputs, which are essentially compressive models that generate outputs efficiently. These models are created in the form of biological recurrent neural networks with a hierarchical structure. The use of these compressive models allows the brain to perceive and interact with the external world, which provides the structure to the phenomenal experience, including self-awareness. Recursion refers here to the cyclic process of creating and refining these models to match the sensory inputs more precisely and perform more efficient compression.

A compressive model is a model of a system or a process that is capable of generating accurate outputs in response to particular inputs with high efficiency. In cognitive systems, these models are formed by the brain as a way of making sense of the sensory data it receives. The models are created using a hierarchy of neurons, and the connections between them are adjusted through a recursive process to better fit the inputs. The use of compressive models by cognitive systems in biological recurrent neural networks plays a vital role in the perception of the external world and provides the structural basis to the phenomenal experience, which includes self-awareness.

As mentioned above, the algorithmic information theory of consciousness presented in [18] is based on the concept of Kolmogorov complexity. Kolmogorov complexity provides a mathematical framework for quantifying the complexity of information. Kolmogorov complexity can conceptually split the Kolmogorov optimal program describing a data string into two parts: a set of bits describing its regularities and another that captures the rest (the part with no structure). The first term is the effective complexity, the minimal description of the regularities of the data. This notion provides the means to account for and separate regularities in data from noise. However, it is generally not possible to calculate the Kolmogorov complexity of an arbitrary string since we have no assurance that the Turing machine (TM) will halt. Despite this, within a limited computation scheme (*e.g.* in terms of computation time or programming language resources), variants of algorithmic complexity, such as Lempel–Ziv–Welch compression, can be calculated. Lempel-Ziv-Welch compression is a simple yet fast algorithm that exploits the repetition of symbol sequences (one possible form of regularity), and its file length is equivalent to the entropy rate, which is an extension of the concept of entropy for stochastic sequences of symbols.

To connect issues of consciousness to those of language with respect to recursion, Roger Vergauwen, in his 'Consciousness, Recursion and Language' [19], examines the role of recursion in the human ability to generate and comprehend language. He argues that recursion is a key factor in the production and comprehension of complex linguistic structures and that it is closely related to the nature of consciousness. According to the author, recursion is an essential part of human cognition and allows for the creation of an infinite variety of novel linguistic expressions.

Vergauwen also discusses the relationship between recursion and consciousness. He argues that the ability to generate and process recursive linguistic structures is a key feature of conscious thought, and that it plays a critical role in the development of abstract thought. He further suggests that the recursive nature of language allowed humans to create symbolic representation and to engage in metacognition. Vergauwen also highlights some of the challenges associated with studying consciousness and recursion and discusses the limitations of current theories of cognitive processing and language acquisition.

Overall, Vergauwen highlights the importance of recursion in human cognition and the role it plays in generating and comprehending complex language. The author makes a case for the close relationship between recursion and consciousness, arguing that this connection is central to our understanding of the nature of human thought and the development of abstract concepts.

Vergauwen discusses several challenges associated with studying consciousness. One of the main issues is the subjective nature of consciousness. Consciousness is a first-person experience that cannot be directly observed or measured by external means. This makes it difficult to develop objective criteria for studying consciousness and to determine what aspects of consciousness are relevant for linguistic processing.

Another significant challenge is the complexity of the neural networks involved in conscious thought. Consciousness arises from the complex interaction of neural processes across multiple regions of the brain. This makes it difficult to isolate specific brain areas or mechanisms that are responsible for generating recursive structures in language. Similarly, it is challenging to identify the specific neural mechanisms or structures that are specifically associated with consciousness.

Finally, Vergauwen notes that the recursive nature of language creates a kind of 'infinite regress,' which poses theoretical challenges for understanding how the brain can process and generate infinitely recursive linguistic structures. This makes it challenging to investigate the neural underpinnings of recursion in language and to develop clear models of how it is processed in the brain. Despite these challenges, the author argues that the study of consciousness and recursion is a critical endeavour for understanding the nature of human thought and for developing a deeper understanding of language and cognitive processing.

According to Vergauwen, current theories of cognitive processing and language acquisition have several limitations when it comes to understanding recursion and consciousness. One limitation is that existing models of linguistic processing often assume a modular and hierarchical organization of the brain, which may not be fully supported by empirical evidence. Hierarchical models of the brain may struggle in accounting for the recursive nature of language processing, especially in cases when recursion is nested infinitely.

Another limitation is that existing theories of language acquisition are often based on the idea of a specific language module, which may not fully capture the complexity of language processing. This specific language module hypothesis may fail to account for the fact that language processing is deeply intertwined with other cognitive processes, such as perception, attention and memory. This may lead to models which do not fully consider the joint effects that human cognitive processes have on language processing.

Finally, Vergauwen points out that current theories of linguistic processing are limited by their over reliance on behavioural and neuroimaging evidence. These restrictions make it difficult to determine the specific neural mechanisms that support the generation and comprehension of recursive structures in language. Vergauwen highlights the need for a multidisciplinary approach that integrates behavioural evidence and neuroscientific evidence from other species, including non-human primates or rodents, to guide our understanding of language processing.

Giulio Tononi with co-authors has presented *Integrated Information Theory (IIT) of consciousness 3.0* [20] that explains how physical mechanisms, such as neurons or logic gates, must be configured to generate experience (phenomenology). The theory defines intrinsic information as 'differences that make a difference' within a system, and integrated information as information specified by a whole that cannot be reduced to that specified by its parts.

According to Integrated Information Theory (IIT), consciousness arises as a result of the causal relationships between a system of mechanisms that are highly integrated and differentiated. The theory posits that conscious experiences are generated by a system with maximum integrated information, or highest Φ value, which characterizes the degree to which the system's parts are mutually dependent and share cause-effect information. A system with high Φ is capable of generating an experience.

Intrinsic information is the information generated by a mechanism over and above the information generated by its individual components, while integrated information is the conceptual information generated by a system of elements over and above the information generated by its minimal parts.

According to Integrated Information Theory (IIT), an experience is identified by a complex of concepts that describe the quality of the experience itself, how much of it there is, and what it is like. These concepts are integrated together among the various parts of the complex, and the more integrated they are, the more unified the experience is. The identity of an experience is then taken to be one complex of concepts, which is irreducible to or different from the other complexes that can be defined by other conceptual decompositions.

Recursion is a central concept in Integrated Information Theory (IIT), which holds that consciousness arises as a result of a system of mechanisms and their causal relationships: 'The idea that "feed-back", "reentry", or "recursion" of some kind may be an essential ingredient of consciousness has many proponents' [20, p. 19]. As described in the article, a complex is considered a mechanism if it generates cause-effect repertoires that are irreducible to those produced by independent parts of the system. These mechanisms can then be combined into larger, more integrated mechanisms, which can be iteratively decomposed into smaller mechanisms again. This recursive organization of mechanisms and their causal relationships is critical for understanding how consciousness arises from the interactions between different parts of the nervous system.

Another recursive account of consciousness [21] defines it as a recursive, spatiotemporal self-location. Consciousness arises as a singular, unified field of recursive self-awareness with explicit orientational characteristics. The psychological structures supporting self-located subjectivity involve an evolutionary elaboration of the two basic elements necessary for extending self-regulation into behavioural interaction with the environment. Understanding consciousness starts with the recognition that cognitive systems serve the biological self-regulatory regime in which they subsist.

The primary reference frame of active waking cognition is *current state* (CS), which is a computation of spatiotemporal self-location updated moment to moment from self-movement feedback (as well as anticipatory *next perceptual state* (FM) of expected feedback). The two basic elements that support self-located subjectivity are an orientative reference frame that structures ongoing interaction in terms of controllable spatiotemporal parameters, and processing architecture that relates behaviour to homeostatic needs via feedback. These two elements are essential for extending self-regulation into behavioural interaction with the environment.

According to the theory, the dedication of cognitive architecture to an anticipative feedforward processing format for speed and energy efficiency implies that the recursive circuitry required to underwrite conscious subjectivity is most likely to have evolved out of such predictive cognitive architecture. Specifically, the theory proposes that evolutionary pressures for ever more energy-efficient sensorimotor processing have encouraged the progressive attenuation of feedforward processing circuitry into a simplified recursive feedforward circuit capable of underwriting autoreferential conscious self-awareness.

The article proposes that consciousness arises through a recursive, spatiotemporal, self-location event schema functioning as a 'self-activated, recursive, working memory circuit' [21, p. 417]. Recursion is an important part of this proposal because the recursive circuitry is what underlies conscious subjectivity by processing anticipatory feedforward predictions together with feedback-derived self-location information to generate and maintain a spatiotemporal model of the self and the environment. Recursion is also discussed as a related concept to hierarchy and attenuation, as it is thought that recursive circuitry emerges out of the attenuation of more complex hierarchical feedforward processing.

Conclusion

The overview of the topic I have managed to present above shows, as it seems, that the initially purely mathematical notion, attempted to be formalized by the greatest minds of the discipline and finally stranded on the shore of ambiguities somewhere between induction and computation, nevertheless represent some important intuition concerning automated enumerating and counting, computations in their broadest sense, language acquisition and mastering and, finally, mind and consciousness. This implies that, being jack of all trades, recursion has no specific traits to define any of these in particular — maybe except for computation, which is disputable, too. But if, as I have conjectured, the recursiveness of all these capabilities is ensured by recurrent loops in the brain, upon which it is weakly supervenient, we have a chance to approach a possible general theory that will explain away all the mentioned capabilities as 'folk-psychological' hypostases of one important capability of computational optimization. Taking into consideration, besides other things, that recurrence is usually taken for a special case of recursion.

This future comprehensive theory will help us crack the code, with which philosophy is inscribed in the book of Universe, and which will lift the veil on our actions lying ahead rejecting 'the absurd tale about free will' (V. I. Lenin).

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